SMALL CANCELATION: EXERCISE SHEET 1

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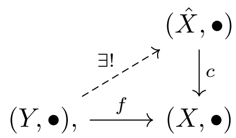
(1) Find a combinatorial definition of the boundary path $P \to \partial D$ of a disk diagram $D \subset \mathbb{R}^2$.
(2) For a based 2-complex $(X, \bullet)$, show that the path homotopy classes of based paths in $(X, \bullet)$ form a group $\pi_1(X, \bullet)$ under concatenation.
(3) Show that a combinatorial map of based 2-complexes $f\colon (X, \bullet) \to (Y, \bullet)$ induces a group homomorphism $f_\#\colon \pi_1(X, \bullet) \to \pi_1(Y, \bullet)$.
(4) Let $H < \pi_1(X, \bullet)$ where $(X, \bullet)$ is a based 2-complex. Construct a based covering map $c\colon (\hat{X}, \bullet) \to (X, \bullet)$ such that $\operatorname{im} c_\# = H$.

    Begin by describing the 0-skeleton $\hat{X}^0$ as equivalence classes of paths $P \to (X, \bullet)$ starting at $\bullet$ where two paths α and β are equivalent if they have the same endpoint and the concatenation $\alpha\bar{\beta}$ of α and the reverse $\bar{\beta}$ of β represents an element of $[\alpha\bar{\beta}] \in H$. Then lift the remaining combinatorial structure.
(5) The Lifting Lemma states that if $c\colon (\hat{X}, \bullet) \to (X, \bullet)$ is a covering map and $f\colon (Y, \bullet) \to (X, \bullet)$ is a combinatorial map with $\operatorname{im} f_\# < \operatorname{im} c_\#$ then there is a unique lift of f to $(\hat{X}, \bullet)$.



    Prove the Lifting Lemma.
(6) Prove that if X satisfies the $C'(\frac{1}{p})$ condition then X satisfies the $C(p+1)$ condition. Does the converse hold?

Date: March 9, 2019.