SMALL CANCELATION: EXERCISE SHEET 3

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(1) Associated to each nonsingular disk diagram D is a nonsingular dual disk diagram D_* which has 1
- one 0-cell for each 2-cell of D and each 1-cell of ∂D,
- one 1-cell for each 1-cell of D and each 0-cell of ∂D, and
- one 2-cell for each 0-cell of D.

Prove that if D satisfies C(p) and T(q) then D_* satisfies C(q) and T(p). After fixing an appropriate orientation on the boundary path of D_*, discover the relationship between turns of D and turns of D_*.

Can you extend the dual D_* and these results to singular disk diagrams?

(2) Notice that T(q) implies T(q') for $q' \leq q$, that C(p) implies C(p') for $p' \leq p$ and that C'(λ) implies C'(λ') for $\lambda' \geq \lambda$. Check that if $\frac{1}{p} + \frac{1}{q} \leq \frac{1}{2}$ then one of
- $(p, q) \geq (6, 3)$, or
- $(p, q) \geq (4, 4)$, or
- $(p, q) \geq (3, 6)$

holds component-wise and that if $\frac{1}{p} + \frac{1}{q} < \frac{1}{2}$ then one of the inequalities holds without equality.

Conclude that if $\frac{1}{p} + \frac{1}{q} = \frac{1}{2}$ then one of D or D_* satisfies C(6)-T(3) or C(4)-T(4) and that if $\frac{1}{p} + \frac{1}{q} < \frac{1}{2}$ then one of D or D_* satisfies C(7)-T(3) or C(5)-T(4).

(3) Prove that any disk diagram D is the union $\bigcup_{i=1}^{k} A_i$ of finitely many subcomplexes A_i such that the following two conditions are satisfied.

(a) Each A_i is either a nonsingular disk diagram or an embedded edge of D^1.

(b) For each $j \in \{1, 2, \ldots, k-1\}$, the intersection $A_{j+1} \cap \bigcup_{i=i}^{j} A_i$ is a single vertex of D.

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Author's Notes

(1) As pointed out by my student, this exercise requires correction. For example, a ladder with at least three 2-cells is C(p) for any p but its dual, as described, is not T(5).