SMALL CANCELATION: EXERCISE SHEET 6

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(1) Let $\frac{1}{p} + \frac{1}{q} \leq \frac{1}{2}$. Let D_+ be an ideal nonsingular C(p)-T(q) disk diagram with a cut 2-cell. Assume that $|D'| \leq |\partial_\circ D'|^2$ for any ideal C(p)-T(q) disk diagram for which $(|D'|, |\partial_\circ D'|) < (|D_+|, |\partial_\circ D_+|)$ lexicographically. Prove that $|D_+| \leq |\partial_\circ D_+|^2$.

(2) Let $\frac{1}{p} + \frac{1}{q} \leq \frac{1}{2}$. Let D_+ be an ideal nonsingular C(p)-T(q) disk diagram without cut 2-cells. Let D' be the union of cells of D_+ that do not intersect ∂D_+. Prove that D' is an ideal C(p)-T(q) disk diagram. Hint: to show that D' is contractible, prove that D_+ deformation retracts onto D'.

(3) Let $\frac{1}{p} + \frac{1}{q} \leq \frac{1}{2}$. Let D_+ be an ideal nonsingular C(p)-T(q) disk diagram without cut 2-cells. Let D' be the union of cells of D_+ that do not intersect ∂D_+. Let A be the completion of $D_+ \setminus D'$ with cell structure pulled back from $A \to D_+$ where $A \to D_+$ is the unique extension of the inclusion $D_+ \setminus D' \hookrightarrow D_+$. Then A is homeomorphic to an annulus. Prove that every cell of A that is not contained in ∂A intersects both components of ∂A.

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