SMALL CANCELATION: EXERCISE SHEET 8

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(1) Prove that in a simply connected C'($\frac{1}{6}$) or C'($\frac{1}{4}$)-$T(4)$ complex, the boundaries of 2-cells are isometrically embedded cycles.

(2) Recall that a presentation $\langle S \mid R \rangle$ is *Dehn* if any trivial word w has a cyclic subword u that is also a cyclic subword of some relator $r \in R$ with $|u| > \frac{1}{2}|r|$. We saw in a previous exercise that if $\langle S \mid R \rangle$ is C'($\frac{1}{6}$) or C'($\frac{1}{4}$)-$T(4)$ then it is Dehn. Using the previous exercise, prove that if $\langle S \mid R \rangle$ is C'($\frac{1}{6}$) or C'($\frac{1}{4}$)-$T(4)$ then for a trivial word w one can obtain u and r as above such that, additionally, $|r| < |w|$.

(3) Find a different family $\{\langle a, b \mid R_S \rangle\}_{S \subset \mathbb{N}}$ of presentations than that of Bowditch given in class such that the taut cycle spectrum $H(\Gamma_S) = qS$ for some $q \in \mathbb{N}$ where Γ_S is the Cayley graph of $\langle a, b \mid R_S \rangle$. Can you choose the R_S so that they do not contain any proper powers?

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