SMALL CANCELATION: EXERCISE SHEET 9

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(1) In the proof of Weinbaum's Theorem B, in the case where property (i) fails we showed in class that K has a projection K_{out} having fewer crossings. Show that if the original projection was alternating then so is that of K_{out}.

(2) Using Weinbaum's Theorem B, prove Weinbaum's Theorem A that a prime, alternating knot has a projection with a C(4)-T(4) Dehn complex.

(3) Let $A_n(a, b) = abab\ldots$ be the alternating word of length n in the alphabet $\{a, b\}$ starting with the letter a. For example $A_2(a, b) = ab$ and $A_3(a, b) = aba$. Let Γ be a finite simplicial graph for which every edge $\{u, v\}$ has a label $m_{\{u,v\}} \in \mathbb{N}_{\geq 2} = \{2, 3, \ldots\}$. The *Artin group* A_Γ defined by Γ is the group with the following presentation.

$$X = \left\langle v \in \Gamma^0 \mid A_n(u, v) = A_n(v, u) \text{ for every edge } \{u, v\} \text{ of } \Gamma \right\rangle$$

Prove that if Γ has no cycle of length 3 then X is C(4)-T(4). Conclude that the Artin groups A_Γ for all such Γ have decidable word and conjugacy problems.

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